

P.5 Factoring

1. Greatest common factor

Greatest common factor - any number and variable that is common to all terms.

$$16x - 24 \quad 8 \text{ is the greatest common factor, so factor out } 8. \text{ (divide each term by } 8)$$
$$8(2x - 3)$$

$$4x^2 - 8x \quad 4x \text{ is the greatest common factor.}$$
$$4x(x - 2)$$

$$12x^4 - 18x^3 + 6x^2 \quad 6x^2 \text{ is the greatest common factor}$$
$$6x^2(2x^2 - 3x + 1)$$

$$x(2x - 3) - 2(2x - 3) \quad (2x - 3) \text{ is the greatest common factor}$$
$$(2x - 3)(x - 2)$$

2. Grouping - 4 terms

$$x^3 - 3x^2 + 4x - 12 \quad \text{Put } () \text{ around the first two terms and around the second two terms. Separate with a plus sign.}$$
$$(x^3 - 3x^2) + (4x - 12) \quad \text{Factor } x^2 \text{ from the first } () \text{ and } 4 \text{ from the second } ().$$
$$x^2(x - 3) + 4(x - 3) \quad \text{Factor out } (x - 3).$$
$$(x - 3)(x^2 + 4)$$

$$x^3 + 6x^2 - 2x - 12 \quad \text{Put } () \text{ around the first two terms and around the second two terms. Separate with a plus sign.}$$
$$(x^3 + 6x^2) + (-2x - 12) \quad \text{Factor } x^2 \text{ from the first } () \text{ and } -2 \text{ from the second } ().$$
$$x^2(x + 6) - 2(x + 6) \quad \text{Factor out } (x + 6).$$
$$(x + 6)(x^2 - 2)$$

$$x^2 - 10a + 5x - 2xa \quad \text{Put } () \text{ around the first two terms and around the second two terms. Separate with a plus sign.}$$
$$(x^2 - 10a) + (5x - 2xa) \quad \text{Since there is no common factor in the first } (), \text{ regroup.}$$
$$(x^2 + 5x) + (-10a - 2xa) \quad \text{Factor } x \text{ from the first } () \text{ and } -2a \text{ from the second } ().$$
$$x(x + 5) - 2a(5 + x) \quad \text{Factor out } (x + 5).$$
$$(x + 5)(x - 2a)$$

3. Trinomials - 3 terms - Trial and error

$x^2 + 8x + 15$
 $(x \quad)(x \quad)$ Since the first term is x^2 , the first term in each () needs to be x .
Now look at the sign on the end number. + means that the signs in the () have to be the same: +, + or -, -. The sign on the middle term indicates both signs must be +.

$(x + \quad)(x + \quad)$ Look at the factors of 15; 1·15 or 3·5. Try each factor.

$(x + 1)(x + 15)$ Check the outer plus the inner.

$15x + 1x = 16x$ We want a middle term of $8x$.

$(x + 3)(x + 5)$

$3x + 5x = 8x$ So the factors are: $(x + 3)(x + 5)$

$2x^2 + 5x - 3$ Since the first term is $2x^2$, the first terms in need to be x and $2x$.

$(2x \quad)(x \quad)$ Now look at the sign on the end number. - means that the signs in the () have to be one + and one -.

$(2x + \quad)(x - \quad)$ The factors of 3 are 3·1.

$(2x + 1)(x - 3)$ Check the outer plus the inner.

$-6x + 1x = -5x$ The number is correct, but the sign is wrong: change the + and -.

$(2x - 1)(x + 3)$

$6x - 1x = 5x$ So the factors are $(2x - 1)(x + 3)$.

$6x^2 - 17x + 12$ Since the first term is $2x^2$, the first terms in need to be $3x$ and $2x$ or $6x$ and x

$(2x - 3)(3x - 4)$ Now look at the sign on the end number. + means that the signs in the () have to be the same: +, + or -, -. The sign on the middle term indicates both signs must be -.

$-8x - 9x = -17x$ Check the outer plus the inner.

4. Difference of 2 Squares (-)

$x^2 - 16$

$(x - 4)(x + 4)$ or $(x + 4)(x - 4)$

$x^2 - 49$

$(x - 7)(x + 7)$

$64x^2 - 81$

$(8x - 9)(8x + 9)$

$x^2 - 3$ will not factor since 3 is not a perfect square.

$x^2 + 4$ will not factor since it is +. Only - will factor with x^2 .

$81x^4 - 1$ any even exponent can be factored as a difference of squares.

$(9x^2 - 1)(9x^2 + 1)$ Factor again.

$(3x - 1)(3x + 1)(9x^2 + 1)$ NOTE: $9x^2 + 1$ will not factor.

5. Perfect Square Trinomials

$$x^2 - 10x + 25$$

$(x - 5)(x - 5)$ Check the outer and the inner.

$$-5x - 5x = -10x$$

$$(x - 5)(x - 5) = (x - 5)^2$$

6. Cubes x^3 (Both + and - will factor)

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

(a) Remove the cubes.

(b) Square the first term.

(c) Opposite sign, then multiply the first term and the second term.

(d) Square the second term. (will always be +)

$$x^3 - 27$$

$$x^3 - 3^3$$

$$(x - 3)(x^2 + 3x + 9)$$

$$8x^3 + 125$$

$$2^3 x^3 + 5^3$$

NOTE: The expression in the second () will never factor.

$$(2x + 5)(4x^2 - 10x + 25)$$

7. **Factor.**

I. Common Factor first if possible.

II. Count the number of terms.

(a) 2 terms x^2 (must have – between the terms) OR x^3 (can factor + or –)

(b) 3 terms - trial and error

(c) 4 terms - group

$$5x^3 - 45x$$

$$5x(x^2 - 9)$$

$$5x(x-3)(x+3)$$

$$6x^2 - 6x - 12$$

$$6(x^2 - x - 2)$$

$$6(x+1)(x-2)$$

$$x^2 + 36$$

Prime (or will not factor)